CALCULATION OF THE ELECTROSTRICTIVE EFFECT IN PRESTRESSED FERROELECTRIC CERAMIC SHELLS UNDER HARMONIC EXCITATION

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When a strong constant electric field acts on a ferroelectric ceramic in the nonpolar phase polarization occurs in the ceramic and a substantially weaker electric field excites harmonic oscillations [1]. The use of an electrostrictive ceramic ensures a linear (anhysteretic) dependence of the mechanical deformations on the electric field, which is important in the construction of adaptive optical systems and micropositioning device [2]. Interest in electrostrictive ceramics has also been stimulated by the possibility of creating stable forms of parametric oscillations [3, 4].

As follows from experiment, when a ferroelectric ceramic is compressed in the direction perpendicular to the polarization vector the electromechanical force factor increases 1.8-2 times. For this reason, as well as in order to expand the range of operating frequencies and to increase the strength, the ferroelectric ceramic is reinforced with metal which prestresses the ceramic. Preliminary mechanical forces of one-third to half the operating load are produced during the fabrication of piezoelectric converters.

We consider a cylindrical shell of ferroelectric ceramic, onto which a thin metal shell is fastened as a result of a temperature difference. A preliminary normal contact pressure $\bar{q}_n$ = const arises between the layers in the process. Alternating electric potentials $\tilde{V} = \tilde{V}_0 \exp(i\omega t)$ ($\bar{V} \gg \tilde{V}$), besides constant potentials $\bar{V}$, are applied to the outer surfaces to the ferroelectric ceramic with coordinates $z = \pm h/2$ (h is the thickness of the shell).

1. The electrostriction equations in terms of the periodic mechanical deformations and electrical quantities have the form [1]

$$
\begin{aligned}
\varepsilon_1 &= s_{11}^E\sigma_1 + s_{12}^E\sigma_2 + s_{13}^E\sigma_3 + 2Q_{12}\bar{E}_3\tilde{E}_3,\\
\varepsilon_2 &= s_{11}^E\sigma_2 + s_{12}^E\sigma_1 + s_{13}^E\sigma_3 + 2Q_{12}\bar{E}_3\tilde{E}_3,\\
\varepsilon_3 &= s_{13}^E(\sigma_1 + \sigma_2) + s_{33}^E\sigma_3 + 2Q_{11}\bar{E}_3\tilde{E}_3,
\end{aligned} \tag{1.1}
$$

where ε_i ($i = 1, 2, 3$) are the strains in the directions of the unit vectors $\boldsymbol{\tau}_1$, $\boldsymbol{\tau}_2$, and $\mathbf{n}$ (see Fig. 1); σ_i are the mechanical stresses; $s_{1i}E$ are the elastic susceptibilities of the ferroelectric ceramic; $E_3 = \bar{E}_3 + \tilde{E}_3$ ($\bar{E}_3$ and $\tilde{E}_3$ are the constant and alternating components of the electric field, $\bar{E}_3 \gg \tilde{E}_3$; and Q_{11} and Q_{12} are the electrostriction constants.

A constant normal contact pressure $\bar{q}_n$ acting on a ferroelectric ceramic shell produces initial compressive, radial $\bar{\sigma}_3$, and circumferential $\bar{\sigma}_2$ stresses, which can be determined by the methods of the two-dimensional theory of elasticity [5]:

$$
\bar{\sigma}_2 = -\frac{\bar{q}_n R}{h}\left(1 + \frac{h}{2R}\right),\ \bar{\sigma}_3 = -\bar{q}_n\left(\frac{1}{2} + \frac{z}{h}\right). \tag{1.2}
$$

Here R is the radius of the middle surface; and $\bar{\sigma}_2$ are the stresses averaged over the thickness. The mechanical stresses $\sigma_i (i = 1, 2, 3)$ consists of constant and variable components $\sigma_i = \bar{\sigma}_i + \tilde{\sigma}_i$ where the variable stresses $\tilde{\sigma}_i$ depend on E_3, $\bar{q}_n$, $\tilde{q}_n$ and are determined here (first the

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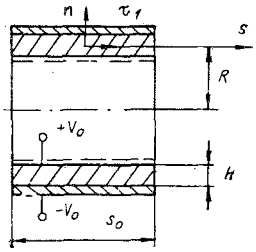

Fig. 1

electrostatic problem, and then the dynamic problem, is solved [6]). Harmonic excitation produces an additional variable normal pressure $\tilde{q}_n$ on the surface in contact with the metal; this pressure depends on the amplitude of the radial displacements and the rigidity of the shell.

We write the circumferential strains of the metal shell as [5] $\tilde{\varepsilon}_2^{(0)} = \tilde{\sigma}_{2M}/E_M = W/R_M$ ($\tilde{\sigma}_{2M}/E_M = W/R_M$ are the dynamic tensile stresses, the elastic modulus, the radial displacements, and the radius of the middle surface of the metal shell). From the relations above if follows that $\tilde{q}_n = E_M h_M W/R_M^2$ for $\tilde{\sigma}_{2M} = \tilde{q}_n R_M/h_M$ (h_M is the thickness of the metal).

From the first two equations of (1.1) we have

$$\begin{gathered}\tilde{\sigma}_1 = \frac{1}{s_{11}^E(1-\mu^2)}(\varepsilon_1 + \mu\varepsilon_2 - \widehat{E}_3),\\ \tilde{\sigma}_2 = \frac{1}{s_{11}^E(1-\mu^2)}(\varepsilon_2 + \mu\varepsilon_1 - \widehat{E}_3),\ \mu = -s_{12}^E/s_{11}^E,\\ \tilde{\sigma}_3 = -\tilde{q}_n(1/2 + z/h),\ \widehat{E}_3 = (1+\mu)(2Q_{12}\overline{E}_3\widetilde{E}_3 + s_{13}^E\tilde{\sigma}_3).\end{gathered} \tag{1.3}$$

The strain distribution along the thickness of the shell is defined by the Kirchhoff–Love hypotheses [5]

$$\varepsilon_1 = \varepsilon_1^{(0)} + z\varkappa_1,\ \varepsilon_2 = \varepsilon_2^{(0)} + z\varkappa_2 \tag{1.4}$$

($\varepsilon_1^{(0)}$ and $\varepsilon_2^{(0)}$ are the strains of the middle surface of the shell, and $\varkappa_1$ and $\varkappa_2$ are the changes in the principal curvatures of that surface).

The relations [7, 8]

$$\overline{E}_3 = \overline{E}_3^{(0)} + z\overline{E}_3^{(1)},\ \widetilde{E}_3 = \widetilde{E}_3^{(0)} + z\widetilde{E}_3^{(1)},\ \overline{E}_3 = -2\overline{V}_0/h,\ \widetilde{E}_3 = -2\widetilde{V}_0/h \tag{1.5}$$

which are analogous to (1.4), can be taken for the electric field strength in the ferroelectric ceramic. The formulas for the constant component $\overline{E}_3^{(1)}$ in the case of electrostatic excitation are given in [6], which also contains $\tilde{E}_3^{(1)}$.

The electric induction D_3 is determined with allowance for (1.1)-(1.5) in terms of the mechanical strain tensor from the expression for the derivative of the internal energy, where electric field strength is the independent variable [1, 9]:

$$\begin{gathered}D_3 = \{\varepsilon_{33}^T - 4Q_{12}(\overline{E}_3^{(0)} + z\overline{E}_3^{(1)})[2Q_{12}(\overline{E}_3^{(0)} + z\overline{E}_3^{(1)})(c_{11}^E + c_{12}^E) +\\ + 2Q_{11}c_{13}^E(\overline{E}_3^{(0)} + z\overline{E}_3^{(1)})] - 2Q_{11}(\overline{E}_3^{(0)} + z\overline{E}_3^{(1)})[4Q_{12}(\overline{E}_3^{(0)} + z\overline{E}_3^{(1)})c_{13}^E +\\ + 2Q_{11}(\overline{E}_3^{(0)} + z\overline{E}_3^{(1)})c_{33}^E]\}(\widetilde{E}_3^{(0)} + z\widetilde{E}_3^{(1)}) + [2Q_{12}(\overline{E}_3^{(0)} + z\overline{E}_3^{(1)})(c_{11}^E + c_{12}^E) +\\ + 2Q_{11}(\overline{E}_3^{(0)} + z\overline{E}_3^{(1)})c_{13}^E][\varepsilon_1^{(0)} + \varepsilon_2^{(0)} + z(\varkappa_1 + \varkappa_2)] +\\ + [4Q_{12}(\overline{E}_3^{(0)} + z\overline{E}_3^{(1)})c_{13}^E + 2Q_{11}c_{33}^E(\overline{E}_3^{(0)} + z\overline{E}_3^{(1)})] \times\\ \times\left\{\frac{s_{13}^E}{s_{11}^E(1-\mu^2)}[(1+\mu)(\varepsilon_1^{(0)} + \varepsilon_2^{(0)}) + (1+\mu)z(\varkappa_1 + \varkappa_2) - 2\widehat{E}_3] -\right.\\ \left. - s_{33}^E\tilde{q}_n\left(\frac{1}{2} + \frac{z}{h}\right) + 2Q_{11}(\overline{E}_3^{(0)} + z\overline{E}_3^{(1)})(\widetilde{E}_3 + z\widetilde{E}_3^{(1)})\right\}.\end{gathered} \tag{1.6}$$

The D_3 distribution is in the nature of the skin effect near the electrodes and in the rest of the shell D_3 is virtually constant over the thickness [8, 10].

Setting the terms with z equal to zero in (1.6), we obtain

$$\widetilde{E}_3^{(1)} = (\overline{\overline{E}}_3 B_1 + \widetilde{E}_3^{(0)} B_2 + \overline{E}_3^{(1)} B_3)/A_1; \quad (1.7)$$

$$A_1 = \varepsilon_{33}^T - B_1(\overline{E}_3^{(0)})^2/2,\ \overline{\overline{E}}_3 = \overline{E}_3^{(0)}\overline{E}_3^{(1)}\widetilde{E}_3^{(0)}, \quad (1.8)$$

$$B_1 = 16Q_{12}[Q_{12}(c_{11}^E + c_{12}^E) + Q_{11}c_{13}^E + (1+\mu)\beta\gamma],$$

$$B_2 = -2(\varkappa_1 + \varkappa_2)[\alpha + \beta\gamma(1+\mu)] + 2\varepsilon_2^{(0)}\frac{E_{\text{M}} h_{\text{M}}}{hR_{\text{M}}}\beta(\gamma s_{13}^E + s_{33}^E),$$

$$B_3 = 2(\varepsilon_1^{(0)} + \varepsilon_2^{(0)})[\alpha + \beta\gamma(1+\mu)] - \varepsilon_2^{(0)}\frac{E_{\text{M}} h_{\text{M}}}{R_{\text{M}}}\beta(\gamma s_{13}^E + s_{33}^E);$$

$$\alpha = Q_{11}(c_{11}^E + c_{12}^E),\ \beta = 2Q_{12}c_{13}^E + Q_{11}c_{33}^E,\ \gamma = \frac{s_{13}^E}{s_{11}^E(1-\mu)}. \quad (1.9)$$

The value of $\overline{E}_3^{(1)}$ is determined from the formulas [6]

$$\overline{E}_3^{(1)} = \frac{E_3^{(0)}[h\overline{A}(\overline{\varkappa}_1) + (\overline{E}_3^{(0)})^2\overline{q}_n s_{33}^E Q_{11}(2Q_{12}c_{13}^E + Q_{11}c_{33}^E)]}{h[\overline{B}(\overline{\varepsilon}_1^{(0)}, \overline{\varepsilon}_2^{(0)}) + (\overline{E}_3^{(0)})^2\overline{C}(\overline{q}_n)]},$$

$$\overline{A} = \overline{\varkappa}_1\left[Q_{12}(c_{11}^E + c_{12}^E) + Q_{11}c_{13}^E + \frac{s_{13}^E(2Q_{12}c_{13}^E + Q_{11}c_{33}^E)}{s_{11}^E(1-\mu)}\right],$$

$$\overline{B}(\overline{\varepsilon}_1^{(0)}, \overline{\varepsilon}_2^{(0)}) = \varepsilon_{33}^T + (\overline{\varepsilon}_1^{(0)} + \overline{\varepsilon}_2^{(0)})[Q_{12}(c_{11}^E + c_{12}^E) + Q_{11}c_{13}^E] + \quad (1.10)$$

$$+ \frac{s_{13}^E}{s_{11}^E(1-\mu)}(2Q_{12}c_{13}^E + Q_{11}c_{33}^E)(\overline{\varepsilon}_1^{(0)} + \overline{\varepsilon}_2^{(0)} + \overline{q}_n s_{13}^E),$$

$$\overline{C}(\overline{q}_n) = -6Q_{12}^2(c_{11}^E + c_{12}^E) - 8Q_{11}Q_{12}c_{13}^E - 3Q_{11}c_{33}^E - 4Q_{12}c_{13}^E -$$

$$- \frac{6s_{13}^E Q_{12}}{s_{11}^E(1-\mu)}(2Q_{12}c_{13}^E + Q_{11}c_{33}^E) + s_{33}^E\overline{q}_n\left(3Q_{11}Q_{12}c_{13}^E + \frac{3}{2}Q_{11}^2 c_{33}^E\right).$$

In (1.10) $\overline{\varkappa}_1$, $\overline{\varepsilon}_1^{(0)}$, and $\overline{\varepsilon}_2^{(0)}$ are found from the electrostatic solution [6].

Substituting (1.4) and (1.5) into (1.3) and expressing the forces T_1 and T_2 at the times M_1 and M_2 in terms of the integrals of $\tilde{\sigma}_1$ and $\tilde{\sigma}_2$ [5], we obtain the following electroelasticity relations:

$$T_1 = D_T(\varepsilon_1^{(0)} + \mu\varepsilon_2^{(0)} - \widehat{E}_3^{(0)}),\ T_2 = D_T(\varepsilon_2^{(0)} + \mu\varepsilon_1^{(0)} - \widehat{E}_3^{(0)}); \quad (1.11)$$

$$M_1 = D_{\text{M}}(\varkappa_1 + \mu\varkappa_2) + M_0,\ M_2 = D_{\text{M}}(\varkappa_2 + \mu\varkappa_1) + M_0, \quad (1.12)$$

$$D_T = \frac{h}{s_{11}^E(1-\mu^2)},\ D_{\text{M}} = \frac{h^3}{12s_{11}^E(1-\mu^2)},\ \widehat{E}_3^{(0)} = (1+\mu)\left(2Q_{12}\overline{E}_3^{(0)}\widetilde{E}_3^{(0)} - \frac{s_{13}^E\widetilde{q}_n}{2}\right),$$

$$M_0 = h^2(s_{13}^E\widetilde{q}_n - 2Q_{12}\overline{E}_3^{(0)}\widetilde{E}_3^{(1)}h)/12s_{11}^E(1-\mu).$$

2. The equations of motion of a cylindrical shell of a ferroelectric ceramic with allowance for the prestressing have the form [11]

$$dT_1^{\text{H}}/d\widetilde{s} - \overline{T}_2^{\text{H}} dW^{\text{H}}/d\widetilde{s} = -\lambda U,\ dN_1^{\text{H}}/d\widetilde{s} - T_2^{\text{H}} - q = -\lambda W^{\text{H}}; \quad (2.1)$$

$$dM_1^{\text{H}}/d\widetilde{s} - \overline{T}_{2(1)}^{\text{H}} dW^{\text{H}}/d\widetilde{s} = N_1,\ \widetilde{s} = s/R, \quad (2.2)$$

$$T_i^{\text{H}} = T_i/D_T\quad (i = 1, 2),\ M_1^{\text{H}} = M_1/RD_T,\ \overline{T}_2^{\text{H}} =$$

$$= \overline{q}_n(2R + h)/(-2D_T),\ \overline{T}_{2(1)}^{\text{H}} = \overline{T}_2^{\text{H}} h/2R,$$

$$U^{\text{H}} = U/R,\ W^{\text{H}} = W/R,\ q = \widetilde{q}_n R/D_T = E_{\text{M}} h_{\text{M}} W^{\text{H}}/D_T,\ \lambda = h\rho\,\omega^2 R^2/D_T,$$

TABLE 1

No. of variant	Excitation frequency, Hz	$\bar{q}_n$, N/m^2	Coefficients of Eq. (2.4)		Deflection $W^H\|_{\tilde{s}=0}$	Right side of Eq. (2.4)
			a_1	a_2		
1	0	10^6	$0{,}269\cdot10^5$	$0{,}29\cdot10^4$	$-0{,}156\cdot10^{-5}$	0
2	50	10^7	$0{,}161\cdot10^4$	$0{,}29\cdot10^4$	$-0{,}195\cdot10^{-4}$	$-0{,}17\cdot10^{-6}$
3	$30\cdot10^3$	0	0,907	$0{,}11\cdot10^4$	$-0{,}308\cdot10^{-3}$	$-0{,}61\cdot10^{-1}$
4	$f_p = 31{,}0\cdot10^3$	0	0,954	$0{,}10\cdot10^4$	$-0{,}258\cdot10^{-2}$	$-0{,}65\cdot10^{-1}$
5	$30{,}0\cdot10^3$	10^6	$0{,}269\cdot10^5$	$0{,}196\cdot10^5$	$-0{,}310\cdot10^{-4}$	$-0{,}611\cdot10^{-1}$
6	$f_p = 31{,}5\cdot10^3$	10^6	$0{,}269\cdot10^5$	$0{,}213\cdot10^5$	$-0{,}192\cdot10^{-3}$	$-0{,}67\cdot10^{-1}$
7	$30{,}0\cdot10^3$	10^7	$0{,}161\cdot10^4$	$0{,}23\cdot10^4$	$-0{,}106\cdot10^{-3}$	$-0{,}61\cdot10^{-1}$
8	$f_p = 32{,}0\cdot10^3$	10^7	$0{,}161\cdot10^4$	$0{,}216\cdot10^4$	$-0{,}15\cdot10^{-3}$	$-0{,}70\cdot10^{-1}$

where s is the linear coordinate in the axial direction; R is the radius of the middle surface of the shell; U and W are the components of the displacement vector of the middle surface in the direction of the unit vectors τ_1 and n (see Fig. 1); $N_1^H = N_1/D_T$ are transverse shear forces; ρ is the density; and $\bar{T}_2^H$ are the prestressing forces.

We express the strains of the middle surface in terms of the displacement [5]

$$\varepsilon_1^{(0)} = dU/ds,\ \varepsilon_2^{(0)} = W/R,\ \varkappa_1 = -d^2W/ds^2 \quad (\varkappa_2 = 0). \tag{2.3}$$

The system of equations (1.11), (1.12), and (2.1)-(2.3) is reduced to the fundamental equation

$$\begin{gathered}
d^6W^H/d\tilde{s}^6 + a_1d^4W^H/d\tilde{s}^4 + a_2d^2W^H/d\tilde{s}^2 + a_3W^H = f,\\
a_1k = d\left(\bar{T}_2^H - \mu_1 + 1\right) - \lambda RD_T - \bar{T}_{2(1)}^H RD_T - a + c - e,\\
a_2k = d\left[\lambda + \mu_1(\lambda - 1) + \bar{T}_2^H(\lambda + 1)\right] +\\
+ RD_T\left[\lambda\left(\bar{T}_{2(1)}^H - 1\right) + (1-\mu)(1+\mu_1) + \mu\bar{T}_2^H\right] +\\
+ E_M h_M R - \lambda(a - c + e),\ k = D_M/R + b,\\
a_3k = \lambda RD_T E_3(Q_{12}),\ E_3(Q_{12}) = 2(1+\mu)Q_{12}\bar{E}_3^{(0)}\tilde{E}_3^{(0)},\\
a = h^3 s_{13}^E E_M h_M/\left(12 s_{11}^E(1-\mu)R\right),\ b = h^3 Q_{12}\left(\bar{E}_3^{(0)}\right)^2 \times\\
\times\left[\alpha + \beta\gamma(1+\mu)\right]/\left(3A_1 R s_{11}^E(1-\mu)\right),\\
c = \frac{h^2 Q_{12}\left(\bar{E}_3^{(0)}\right)^2 E_M h_M \beta\left(\gamma s_{13}^E + s_{33}^E\right)}{3R_M A_1 s_{11}^E(1-\mu)},\ d = \frac{h^3 Q_{12}\bar{E}_3^{(0)}\bar{E}_3^{(1)}}{3A_1 s_{11}^E(1-\mu)}\left[\alpha + \beta\gamma(1+\mu)\right],\\
e = \frac{h^3 Q_{12}\bar{E}_3^{(0)}\bar{E}_3^{(1)} E_M h_M \beta\left(\gamma s_{13}^E + s_{33}^E\right)}{6A_1 s_{11}^E(1-\mu)R_M},\ fk = \lambda RD_T E_3(Q_{12}),\\
\mu_1 = \mu + \frac{s_{13}^E E_M h_M}{2R_M}(1+\mu).
\end{gathered} \tag{2.4}$$

The constants α, β, γ, and A_1 are determined by Eqs. (1.8)-(1.10). For the case of axisymmetric free oscillations of the shell we adopt the boundary-value conditions

$$M_1^H\Big|_{\tilde{s}=\tilde{s}_0/2} = N_1^H\Big|_{\tilde{s}=\tilde{s}_0/2} = T_1^H\Big|_{\tilde{s}=\tilde{s}_0/2} = 0. \tag{2.5}$$

Corresponding to the first two conditions of (2.5) are the equations

$$\begin{gathered}
-\frac{D_M}{R}\frac{d^2W^H}{d\tilde{s}^2} + \frac{h^2 s_{13}^E E_M h_M W^H}{12R_M s_{11}^E(1-\mu)} = \frac{h^3 Q_{12}\bar{E}_3^{(0)}\tilde{E}_3^{(1)}}{6s_{11}^E(1-\mu)},\\
-\frac{D_M}{R}\frac{d^3W^H}{d\tilde{s}^3}\left[\frac{h^3 s_{13}^E E_M h_M}{12R_M s_{11}^E(1-\mu)} - \bar{T}_{2(1)}^H\right] = \frac{h^3 Q_{12}\bar{E}_3^{(0)}}{6s_{11}^E(1-\mu)}\frac{d\tilde{E}_3^{(1)}}{d\tilde{s}}.
\end{gathered}$$

The changes in the curvature $\varkappa_1$ and strains $\varepsilon_1^{(0)}$ and $\varepsilon_2^{(0)}$ in the case of electrostatic loading are virtually constant along the length of the cylindrical shell; hence we assume $\bar{E}_3^{(1)}$ to be constant in the expression (1.7) for $\tilde{E}_3^{(1)}$ and the function $d\tilde{E}_3^{(1)}/d\tilde{s}$ from (1.7) is determined from

$$A_1 \frac{d\tilde{E}_3^{(1)}}{d\tilde{s}} = \bar{E}_3^{(0)} \Big\{ -2 \frac{d\tilde{\varkappa}_1}{d\tilde{s}} [\alpha + \beta\gamma(1+\mu)] + 2 \frac{d\tilde{\varepsilon}_2^{(0)}}{d\tilde{s}} \frac{E_M h_M}{R_M h} \beta\left(\gamma s_{13}^E + s_{33}^E\right) \Big\} + \bar{E}_3^{(1)} \Big\{ 2 \left(\frac{d\tilde{\varepsilon}_1^{(0)}}{d\tilde{s}} + \frac{d\tilde{\varepsilon}_2^{(0)}}{d\tilde{s}} \right) [\alpha + \beta\gamma(1+\mu)] - \frac{d\tilde{\varepsilon}_2^{(0)}}{d\tilde{s}} \frac{E_M h_M}{R_M} \beta\left(\gamma s_{13}^E + s_{33}^E\right) \Big\}. \tag{2.6}$$

Here the constant A_1 is found from (1.8). From (1.11) with allowance for (2.3) we express T_1^H in terms of the displacements

$$T_1^H = dU^H/d\tilde{s} + \mu_1 W^H - E_3(Q_{12}),\ E_3(Q_{12}) = 2Q_{12}(1+\mu)\bar{E}_3^{(0)}\tilde{E}_3^{(0)}. \tag{2.7}$$

From the boundary condition $T_1^H(\tilde{s} = \tilde{s}_0/2) = 0$ with allowance for (2.3) and (2.7) it follows that $d^2U^H/d\tilde{s}^2 = -\mu_1 dW^H/d\tilde{s}$, for $\tilde{s} = \tilde{s}_0/2$ and (2.6) takes on the form

$$A_1 \frac{d\tilde{E}_3^{(1)}}{d\tilde{s}} = 2\bar{E}_3^{(0)} \left\{ \frac{1}{R} \frac{d^3W^H}{d\tilde{s}^3} [\alpha + \beta\gamma(1+\mu)] + \frac{dW^H}{d\tilde{s}} \frac{E_M h_M}{R_M h} \beta\left(\gamma s_{13}^E + s_{33}^E\right) \right\} + \bar{E}_3^{(1)} \frac{dW^H}{d\tilde{s}} \left\{ 2(1-\mu_1)[\alpha + \beta\gamma(1+\mu)] - \frac{E_M h_M}{R_M} \beta\left(\gamma s_{13}^E + s_{33}^E\right) \right\}.$$

The last boundary condition of (2.5) gives a third equation for determining the constants of integration:

$$-\frac{(D_M + bR)}{R} \frac{d^4W^H}{d\tilde{s}^4} + \left[a - c - d + e - \bar{T}_{2(1)}^H R D_T - d\left(\bar{T}_2^H - \mu_1\right)\right] \frac{d^2W^H}{d\tilde{s}^2} + \{2RD_T(\lambda - 1 + \mu\mu_1) - E_M h_M [s_{13}^E D_T(1+\mu) - 2R] - 2\lambda d\mu_1\} W^H = [(\mu - 1) RD_T - \lambda d] E_3(Q_{12}).$$

The solution (2.4) is obtained from an analysis of the roots of its characteristic equation

$$x^6 + a_1 x^4 + a_2 x^2 + a_3 = 0. \tag{2.8}$$

By means of the successive exchanges $x = y_1^{1/2}$, $y_1 = y - a_1/3$ we find the cubic equation $y^3 + py + q = 0$, $p = a_2 - a_1^2/3$, $\hat{q} = a_3 - a_1 a_2/3 + 2a_1^3/27$ from (2.8).

When the parity and symmetry of W^H relative to the origin $\tilde{s} = 0$, located at the center of the cylindrical shell (see Fig. 1), are taken into account one solution of (2.4) at given relations of the coefficients $a_i (i = 1, 2, 3)$ has the form [12]

$$W^H = C_1 \mathrm{ch}(x_1\tilde{s}) + C_2 \mathrm{ch}\bar{\alpha}\tilde{s}\cos\bar{\beta}\tilde{s} + C_3 \mathrm{sh}\bar{\alpha}\tilde{s}\sin\bar{\beta}\tilde{s} + \frac{f}{a_3},$$
$$\bar{\alpha} = \frac{\bar{b}}{2\bar{\beta}},\quad \bar{b} = \frac{\bar{\bar{A}} - \bar{\bar{B}}}{2}\sqrt{3},\quad \hat{b} = \left(-\frac{\bar{a}}{2} + \frac{\sqrt{\bar{a}^2 + \bar{b}^2}}{2}\right)^{1/2},$$
$$\bar{a} = -(\bar{\bar{A}} + \bar{\bar{B}})/2,\ \bar{\bar{A}} = (-\hat{q}/2 + \sqrt{D})^{1/3},\ \bar{\bar{B}} = (-\hat{q}/2 - \sqrt{D})^{1/3},$$
$$D = (p/3)^3 + (\hat{q}/2)^2, \tag{2.9}$$

where x_1 is the first root of the characteristic equation [in the case of negative radicand $x = (y - a_1/3)^{1/2}$ the function $\cosh|x_1|\tilde{s}$ instead of $\cosh x_1\tilde{s}$, and at other values of $\hat{\alpha}$ and $\hat{\beta}$ the solution (2.9) is expressed in terms of hyperbolic instead of trigonometric functions].

For a shell made of a ceramic of the TsTSL type [reinforced with a metal layer of thickness $h_M = 0.2$ m, having an elastic modulus $E_M = 2.1 \cdot 10^{11}$ N/m^2], with the geometric parameters $R = 16$ mm, $h = 3$ mm, $s_0 = 32$ mm, and $Q_{12} = -1.3 \cdot 10^{-16}$ m^2/V^2 at $\overline{V} = 1.5$ kV and $\tilde{V} = 300$ V the results of calculations at various frequencies and values of $\overline{q}_n$ are given in Table 1, from which we see that prestressing force $\overline{q}_n$ only slightly increases the resonance frequency of the metal-ceramic shell and in the case under consideration decreases its displacement W^H by an order of magnitude (variants 4, 6, and 8). For variant 2 (at 50 Hz) the derivatives of W^H are $d^2W^H/d\tilde{s}^2 = 3.95 \cdot 10^{-6}$, $d^4W^H/d\tilde{s}^4 = -5.51 \cdot 10^{-13}$, and $d^6W^H/d\tilde{s}^6 = -4.72 \cdot 10^{-11}$. A comparison of the derivatives, the coefficients, and the right side of Eq. (2.4) indicates that the fourth and sixth derivatives can be disregarded at low frequencies.

LITERATURE CITED

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